Statement of Research Results

Antonio Palacios, Professor
Mathematics Department
Computational Science and Engineering Research Center
San Diego State University

Over the past eighteen years I have developed an interdisciplinary undergraduate and graduate research program in *Applied and Computational Mathematics*. The work has an emphasis on combining dynamical systems theory, group theoretical methods, i.e., equivariant bifurcation theory and representation theory, and computational methods to model, analyze, and predict the behavior of ***Complex Nonlinear Systems***.

At the beginning of my career, I was interested in pattern formation and worked on PDEs and ODEs to study, numerically and analytically, the symmetry-breaking bifurcations that give rise to cellular flame patterns. I was also interested in *Mathematical Biology* and worked with Prof. Marty Golubitsky (current director of the Mathematical Biosciences Institute at Ohio State)) on the existence and stability of heteroclinic connections in coupled cell systems. I then extended that work to study the rhythms of the heart beat of medicinal leeches. In the 2000's, my work focused on applications to engineering and physics with the *fundamental principle* of mimicing what nature does best: to create networks of tightly interconnected artificial cells, i.e., devices, with collective optimal responses. In collaboration with engineers and experimentalists, we worked on *deterministic* and *stochastic* models of: biologically-inspired networks of magnetic and electric field sensors; 2D arrays of superconducting quantum loops; networks of vibratory gyroscopes; and arrays of nano-oscillators. In the past few years, previous works have led us into new topics: networks of energy harvesting systems; networks of Hamiltonian cells; computational methods to simulate and characterize Ozone concentrations and related pollutants from PDEs; and analysis of physiological measurements. These, and other related projects have been funded by various federal agencies: ARO, NSF, DOE, DoD, ONR, and NSA. Below is a brief description of some representative projects.

Biologically-inspired Network Sensor Devices

A major theme of my research is to explore the advantages exhibited by biological network systems to model, design, and fabricate, *sensory devices* that can harness cooperative behavior to increase sensitivity at low power. For instance, sharks can sense extremely weak electric fields due to the collective output of thousands of detectors called *ampullae of Lorenzini*. A general question is to investigate what aspects of the network dynamics are due to *network topology* and what aspects are due to details of the internal dynamics of each cell. In collaboration with scientists and engineers from the Space and Naval Warfare Center, San Diego, we have modeled, analyzed and fabricated a new generation of highly-sensitive (up to 400 times more sensitive than conventional sensors) networks of magnetic and electric field sensors. The network exploits the *bistability* of each unit and the topology of *heteroclinic cycles* induced by cyclic symmetry. The work has been supported by NSF, ONR, DoD, and has lead to internships for participating students, manuscripts, and multiple U.S. patents, please see vitae. We are currently developing the concept of a Universal Ultra Sensitive Network System capable of detecting: magnetic field, electric field, seismic, acoustic, infrared, thermal neutrons, chemical and biological multi-agent sensing (CMAS), and other signals in one single unit, see Fi.g 1.

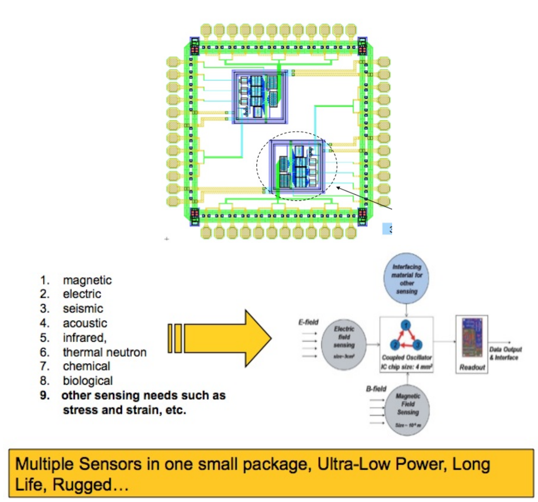


Figure 1: Universal ultra-sensitive sensor.

MEMS Networks of Vibratory Gyroscopes

Funded by the National Science Foundation, we have shown, analytically and computationally, proof of concept that networks of coupled gyroscopes, modeled by

$$\begin{aligned} m\ddot{x}_j + c\dot{x}_j + \kappa x_j + \mu x_j^3 &= A_d \sin w_d t + 2m\Omega_z \dot{y}_j + \sum_{k\to j} c_{jk} h(x_j, x_k) \\ m\ddot{y}_j + c\dot{y}_j + \kappa y_j + \mu y_j^3 &= -2m\Omega_z \dot{x}_j, \end{aligned}$$

can offer significant performance improvements by reducing the negative effects of noise while minimizing phase drift. We also developed a novel scheme, a *drive-free* network-based gyroscope system, in which the coupling topology can lead to self-regulated limit cycle oscillations in the drive- and sense-axes with stable constant amplitude and phase-locking. This work was selected and advertised by AIP (American Institute of Physics) as a research highlight due to its originality and novelty of the concept. The next step is to explore a possible MEMS realization of the models. Current and future work include a collaboration with Prof. Takashi Hikihara from Kyoto University to fabricate and test a miniaturized version, via MEMS technology, of a network of vibratory gyroscope system. In addition, the work has led to many unanswered questions regarding the nature of the bifurcations in large networks. A perturbation analysis is not well suited, so we are currently investigating a different approach via coupled Hamiltonian systems.

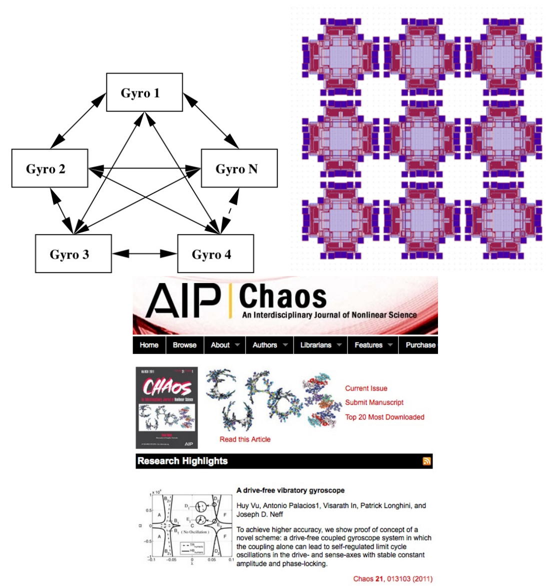


Figure 2: Network of Vibratory Gyroscopes.

Complexity on Human and Engineered Systems: Energy Harvesting

Energy harvesting devices are key to a wide range of technologies: wireless sensor networks, micro wind turbine, thermoelectric generators, health monitoring systems, human-generated (see Fig. 3) power devices for biomedical applications and, of course, consumer products such as automatic wristwatches and TV remote controls, to the cloud. A major challenge to advance these technologies is the fact that the amount of electrical energy produced is small for these devices. To circumvent this problem, several groups have considered nonlinear materials and have proposed innovative designs and techniques, which together can improve power output. The improvements are, however, reaching the fundamental limits of power output that can be harvested with single units. Only in recent years networks of multiple harvesters have started to attract attention but a comprehensive analysis of the governing equations:

$$\begin{aligned} m\frac{d^2 z_j}{dt^2} + b\frac{dz_j}{dt} + k_1 z_j + k_3 z_j^3 &+ \sum_{k\to j} c_{jk} h_z(z_j, z_k) &= F_e(t) - Gi_T \\ L_c \frac{di_j}{dt} + (R_L + R_c) i_j &+ \sum_{k\to j} m_{jk} h_i\left(\frac{di_j}{dt}, \frac{di_k}{dt}\right) &= G\frac{dz_j}{dt}, \end{aligned}$$

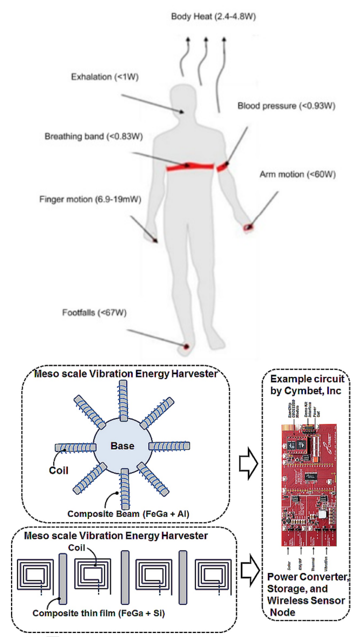


Figure 3: Human power.

has not been carried out yet. This is a more recent project in which I am currently modeling and analyzing the power output response of a network of energy harvesters made up of magnetorestrictive materials. One of the goals is to determine scaling laws for the energy output as a function of the size of the network and the topology of connections.

Networks of Nano-Oscillators for Microwave Signals

The 2007 Nobel prize in Physics was awarded jointly to Albert Fert and Peter Grunberg for their discovery of the Giant Magnetoresistance Effect (GMR). A spin-polarized current can exert a torque on the magnetization of a ferromagnetic layer leading to precession. Then the GMR effect can convert the magnetic precession into microwave voltage signals and turn the valve into a *Spin Torque Nano-Oscillator* (STNO) but its power output, about 1 nW, is too small for practical applications. A possible solution, is to synchronize a network of STNOs so that a coherent signal with a common frequency and phase can be extracted from the network to produce a more powerful microwave signal. Supported by NSF, I am currently studying and classifying the various coherent states that a network of STNOs can produce, finding conditions for the existence and stability of such coherent states, determining the effects of different couplings and connection topologies, through the Landau-Lifshitz-Gilbert-Slonczewski equation:

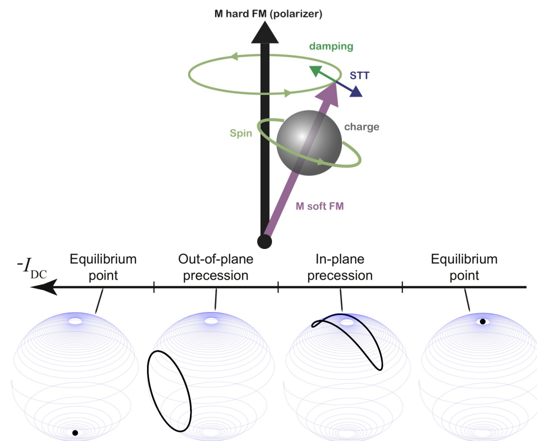


Figure 4: Spin Torque Nano-oscillator.

$$\frac{dm_{\text{free}}}{dt} = \overbrace{-m_{\text{free}} \times h_{\text{eff}}}^{\text{precession}} - \overbrace{\alpha m_{\text{free}} \times (m_{\text{free}} \times h_{\text{eff}})}^{\text{LL damping}} + \overbrace{\gamma m_{\text{free}} \times (m_{\text{free}} \times m_{\text{fixed}})}^{\text{spin torque}}.$$

Networks of Superconductive Loops

Supported by NSA Tactical SIGINT Technology Program, we have conducted extensive analysis and computer simulations of the collective voltage response of networks of thousands of superconducting quantum loops, modeled by

$$\dot{\psi}_{i,j} = \mathbf{F}(\psi_{i,j}, \lambda),$$

where $\psi_{i,j}$ are the phases of the Josephsons Junctions in the network. The aim is the development of a sensitive, low noise, significantly lower Size, Weight and Power (SWaP) antenna, capable of meeting all requirements for certain class ships for Information Operations/Signals Intelligence (IO/SIGINT) applications in Very High Frequency/Ultra High Frequency (V/UHF) bands. The device will increase the listening capability of receivers by moving technology into a new regime of energy detection allowing wider band, smaller sized, more sensitive, more stealth systems. The smaller size and greater sensitivity will allow for ships to be de-cluttered of their current large dishes and devices. Ongoing work includes derivation of asymptotic analytical approximations to the exact solutions. Such solutions could be used to expedite future analysis of other array configurations. Future work also includes a study of the existence and stability of **chimera states** in networks of superconducting loops. A chimera state is a peculiar pattern of collective behavior of an array of oscillators in which a cluster, smaller than the entire network of oscillators, is in a coherent state, possibly phase-locked, while simultaneously another group is in an incoherent state, i.e., their phase dynamics exhibits chaos. We conjecture that chimera states can lead, under certain conditions, to an alternative device via nonlocal magnetic coupling without the need to hard-wire coupling of the superconducting loops.

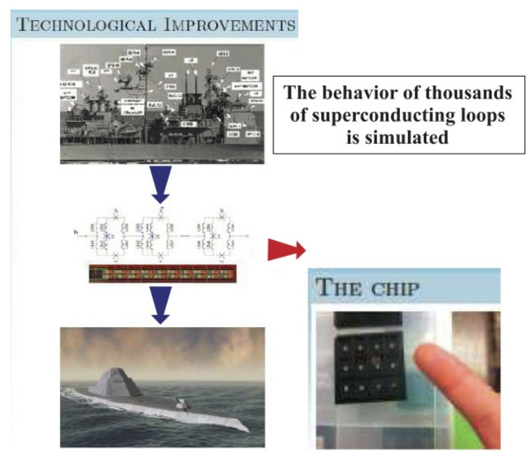
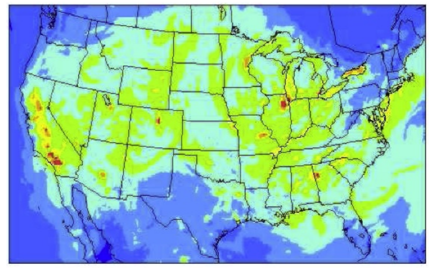


Figure 5: Network of Superconducting Loops.

Computational Methods to Study Ozone Formation



Computer simulations of ozone concentrations

Tropospheric ozone is deleterious to both human and ecosystem health. It is produced through *non-linear chemical reactions* involving precursor emissions of nitrogen oxides (NO_x) and volatile organic compounds. Comprehensive air pollution models include detailed treatment of the emissions, transport (advective, turbulent, clouds), and chemistry dictating the spatial and temporal distribution of O_3 and related precursor species. From a policy standpoint a question of interest is to determine how much an individual source contribute to the O_3 at a given location and time. While many techniques ranging from simple brute-force methods to formal sensitivity analysis have been used, the "backward" attribution of O_3 at a given location/time to its sources remains challenging due to the non-linearities in the system. In this project we will address the question of determining the source of O_3 emission by solving an inverse problem from a Partial Differential Equation model for ozone concentration:

$$\frac{\partial \mathbf{C}}{\partial t} = -\nabla(\mathbf{uC}) + \nabla(\mathbf{K}\nabla\mathbf{C}) + \mathbf{R} + \mathbf{E} + \ldots, \tag{1}$$

where $\mathbf{x}$ is a grid defined over a given domain, e.g., the continental United States, $\mathbf{u}(\mathbf{x}, t)$ is the wind field, $\mathbf{K}(\mathbf{x}, t)$ is the turbulent diffusivity tensor, $\mathbf{R}_i(\mathbf{x}, t)$ are the net rates of chemical production, $\mathbf{E}_i(\mathbf{x}, t)$ are the emission rates produced by multiple sources $\mathbf{S}_i(\mathbf{x}, t)$, and the ellipsis denote other processes. Eq. (1) will be solved using operator splitting methods and the Proper Orthogonal Decomposition will be used to find spatial identifiers associated with individual emission sources. Image processing techniques and data analytics will be performed, perhaps with the aid of High Performance Computing resources.

Data Analytics: Understanding Human and Social Behavior

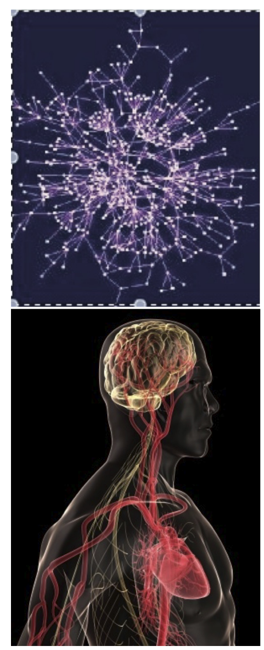


Figure 6: Brain Heart Networks.

By and large, the most complicated of all networks with nonlinear input/output characteristics is the brain. In this system, nonlinear analysis of physiological data represents a significant change in the way in which scientists approach the understanding of human behavior. Although nonlinear analysis has had some success with identifying pathological conditions, concepts such as *trust* and *distrust* do not have the same clear ground truth available from postmortem analysis. However, for DoD tactical actions, intelligence gathering, and command and control, the ability to define and identify an objective measure of trust or distrust has the potential to save both lives and money. It would also be a valuable tool during peacetime for security applications, and could potentially be so far-reaching as to impact educational methods, corporate leadership, and treatment of conditions such as Post-Traumatic Stress Disorder or PTSD. The work proposed here starts with a preliminary outline of the nonlinear methods to analyze a single physiological measurement: *electrocardiogram* (ECG) data, in particular, cardiac interbeat interval (IBI). Preliminary analysis suggests that there are two distinct attractors. One associated with TRUST and one with DISTRUST. Each attractor has a distinguishable Lyapunov Exponent Spectrum. Further study, such as increasing the number of subjects or investigating other physiological metrics, is warranted. The results described so far present an intriguing possibility; if correlates for trust exist in heart rate variability, do they exist in other physical measures? During the acquisition of the ECG signals, galvanic skin response (GSR) and electroencephology (EEG) signals have also been recorded. These signals, especially the EEG recordings, may provide further evidence for physiological correlates of human trust.

Future Work

The works outlined above have led to many interesting issues that need to be investigated, analytically and computationally. Analytically, it turns out that the governing equations for many of the examples discussed above, e.g., gyroscopes, energy harvesters or superconducting loops, can be treated as perturbation of Hamiltonian systems. Analysis of the collective dynamics via Hamiltonian mechanics can be quite useful to uncover the nature of the bifurcations in high-dimensional networks. However, there are significantly fewer results for symmetric networks with Hamiltonian structure. Computationally, all of the problems described above require extensive computational resources. For instance, the Ozone formation inverse problem can handily keep a peta- or exaflop environment busy for days or weeks at a time; depending on the number of chemical species in the simulation. The energy harvesting networks may also lead to large systems of stochastic differential equations. And the analysis of ECG and EEG data may require developing new algorithms for the analysis of large data sets.