$$\frac{\partial u}{\partial t} = \left[\mu - (\nabla^2 + k_c^2)^2\right] u - u^3$$

BC: $u = \frac{\partial^2 u}{\partial x^2} = 0, \quad x = 0, L$

Trivial sol: $u_0 = 0$

$$\frac{1}{G}\frac{dG}{dt} =$$

Linearize around $u_0 = 0$:

$$\frac{\partial \tilde{u}}{\partial t} = \left[\mu - (\nabla^2 + k_c^2)^2\right] \tilde{u} = \left[\mu - (\nabla^4 + 2k_c \nabla^2 + k_c^2)\right] \phi(x) G(t)$$

Let $\tilde{u}(x,t) = G(t)\,\phi(x) \Rightarrow \frac{\phi(x)}{G(t)}\frac{dG}{dt} = \left[\mu - (\nabla^2 + k_c^2)^2\right]\phi(x)$

$$\phi(x)\frac{dG}{dt} = -\left[(\nabla^2 + k_c^2)^2 - \mu\right] G(t)\phi(x)$$

$$\frac{1}{G(t)}\frac{dG}{dt} = -\frac{1}{\phi(x)}\left[(\nabla^2 + k_c^2)^2 - \mu\right]\phi(x) = \sigma$$

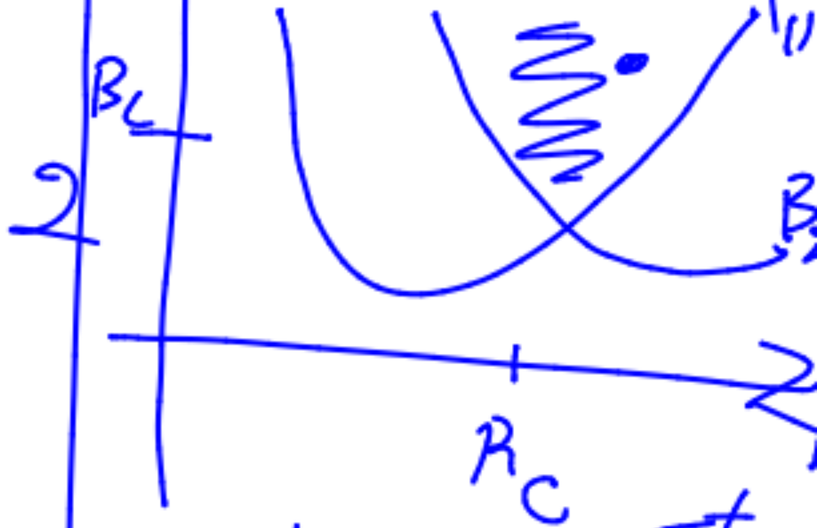
$$\boxed{\frac{dG}{dt} = \sigma G(t)} \Longrightarrow G(t) = C e^{\sigma t}$$

$$-\left[(\nabla^2 + k_c^2)^2 - \mu\right]\phi(x) = \sigma\,\phi(x)$$

$$\boxed{(\nabla^2 + k_c^2)^2\phi(x) + (\sigma - \mu)\phi(x) = 0, \quad \begin{array}{l}\phi''(x=0) = 0\\ \phi''(x=L) = 0\end{array}}$$

$$\frac{\partial u}{\partial t} = k_1 \nabla^2 u + (B-1)u + A^2 v - \eta_1 u^3 - \nu_1 (\nabla u)^2$$

$$\frac{\partial v}{\partial t} = k_2 \nabla^2 v - Bu - A^2 v - \eta_1 u^3 - \nu_2 (\nabla v)^2$$




$e^{\sigma t}\psi_{21} + e^{\sigma t}\psi_{1}$

$(u_0, v_0) = (0,0)$. Let $z = (u, v)$

$$z_t = J z + D \nabla^2 z, \quad J = \begin{bmatrix} B-1 & A^2 \\ -B & -A^2 \end{bmatrix}, \quad D = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

Let $z(x,y,t) = e^{\sigma t + i \vec{k} \cdot \vec{x}}$

$$\Rightarrow \nabla^2 z + k^2 z = 0, \quad |\vec{k}| = k \Rightarrow k^2 = (\alpha_{nm}/R)^2$$

$$\big| \underbrace{J - k^2 D - \sigma I}_{(*)} \big| = 0 \Rightarrow \sigma^2 - tr(*)\sigma + det(*) = 0$$

$$\xrightarrow[\sigma = 0]{\text{marginal stability}} k_1 k_2 k^4 + \left[k_1 A^2 - k_2 (B-1)\right] k^2 + A^2 = 0$$

$$\Rightarrow \boxed{B_{nm} = 1 + \frac{k_1}{k_2} A^2 + k_1 \left(\frac{\alpha_{nm}}{R}\right)^2 + \frac{A^2}{k_2}\left(\frac{R}{\alpha_{nm}}\right)^2}$$

Hopf Bif.

e.g.) $\dot{r} = \mu r - r^3$, $\dot{\theta} = \omega + b r^2$ $\overset{x = r\cos\theta}{\underset{y = r\sin\theta}{\Longleftrightarrow}}$ $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{bmatrix} \mu & -\omega \\ \omega & \mu \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} + \text{h.o.t.}$

$\text{e-vals}(A) = \mu \pm \omega i$

$$\underbrace{\boxed{\begin{aligned} \dot{r} &= d\mu r + a r^3 \\ \dot{\theta} &= \omega + c\mu + b r^2 \end{aligned}}}_{NF}$$

Lemma 1 For $-\infty < \frac{\mu d}{a} < 0$ and $\mu$ is sufficiently small then

$$(r(t), \theta(t)) = \left(\sqrt{-\frac{\mu d}{a}},\ \left[\omega + \left(c - \frac{bd}{a}\right)\mu\right] t + \theta_0\right)$$

is a periodic sol. of NF.

## Lemma 2

The periodic sol. is

(i) Asymptotically stable for $a<0$ (supercritical)

(ii) Unstable for $a>0$ (subcritical)

Th: Center Manifold Reduction