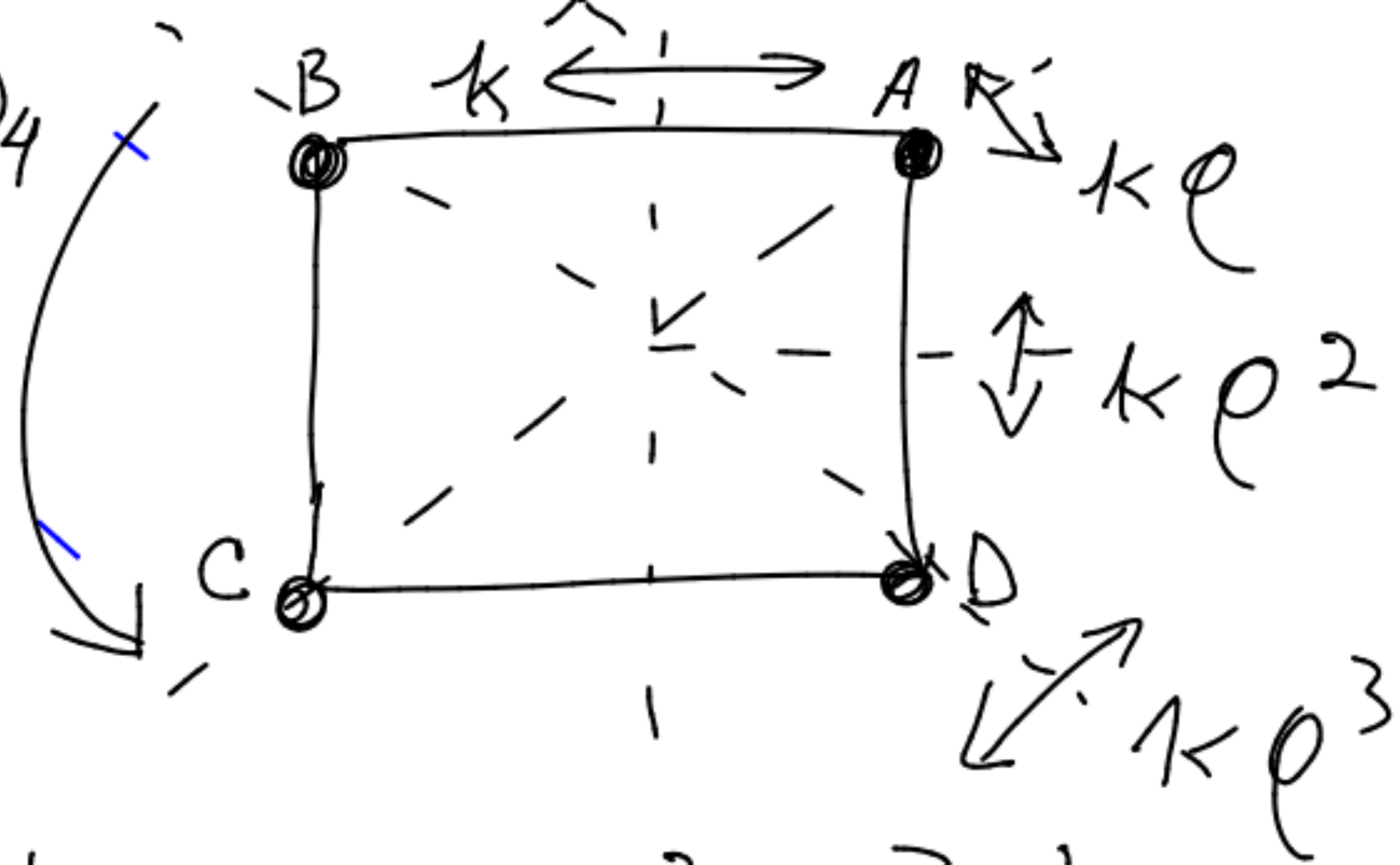
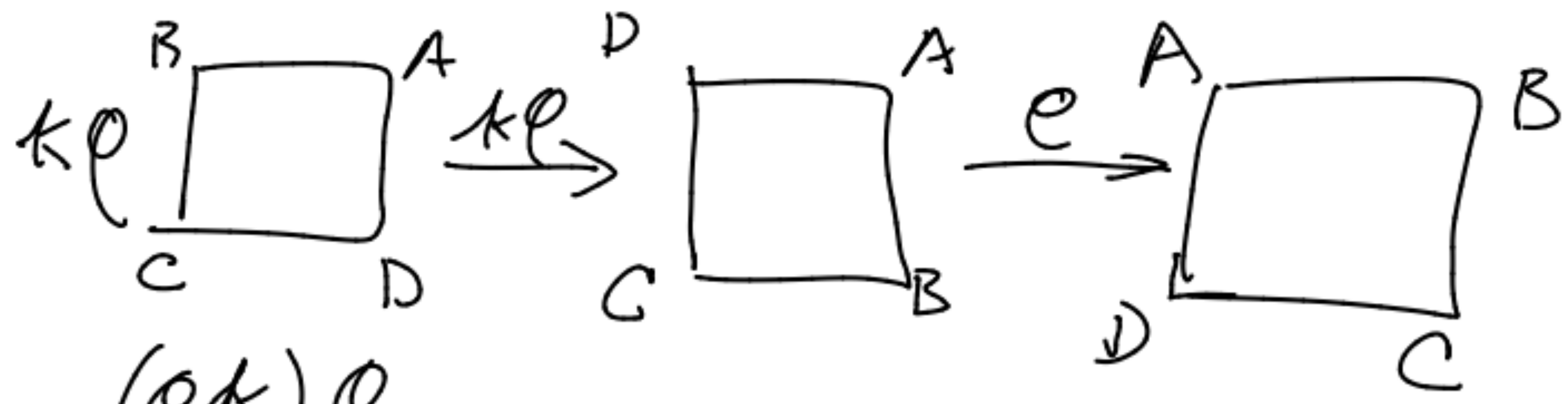
3.2 D_4

$\frac{2\pi}{4} \approx \rho$



$\rho \cdot k\rho \cdot (z = a+bi)$

3.4



$(\rho k)\rho$

	e	ρ	ρ^2	ρ^3	k	$k\rho$	$k\rho^2$	$k\rho^3$
e	e	ρ	ρ^2	ρ^3	k	$k\rho$	$k\rho^2$	$k\rho^3$
ρ	ρ	ρ^2	ρ^3	e		k		
ρ^2	ρ^2	ρ^3	e	ρ				
ρ^3	ρ^3	e	ρ	ρ^2				
k	k							
$k\rho$	$k\rho$							
$k\rho^2$	$k\rho^2$							
$k\rho^3$	$k\rho^3$							

Axioms

$\gamma_1, \gamma_2 \in \Gamma$

$(\gamma_1\gamma_2)\gamma_3 = \gamma_1(\gamma_2\gamma_3)$

$\gamma e = e\gamma = \gamma$

$\gamma^{-1}\gamma = \gamma\gamma^{-1} = e$

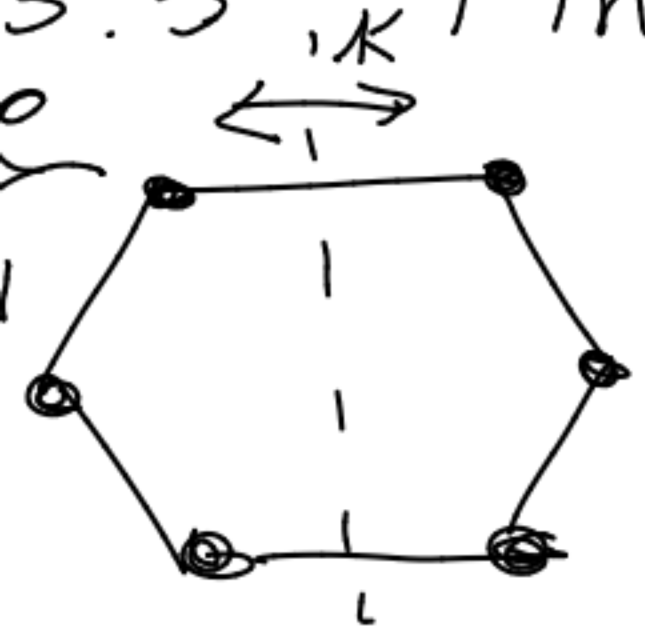
.4 – Find subgroups of D_4

– Find normal subgroups.

subgroups: $H = \{\{e\}, \{e, k\}, \{e, \rho, \rho^2, \rho^3\}, \dots_{10}\}$

$N_{ormal}(H) = \{ \qquad H\gamma = \gamma H \}$

3.5 Find Normalizers of $D_2, Z_2 \subset D_6$



$\underset{\text{Normalizer}}{N}(D_2) = \{\gamma \in D_6 : D_2\gamma = \gamma D_2\}$

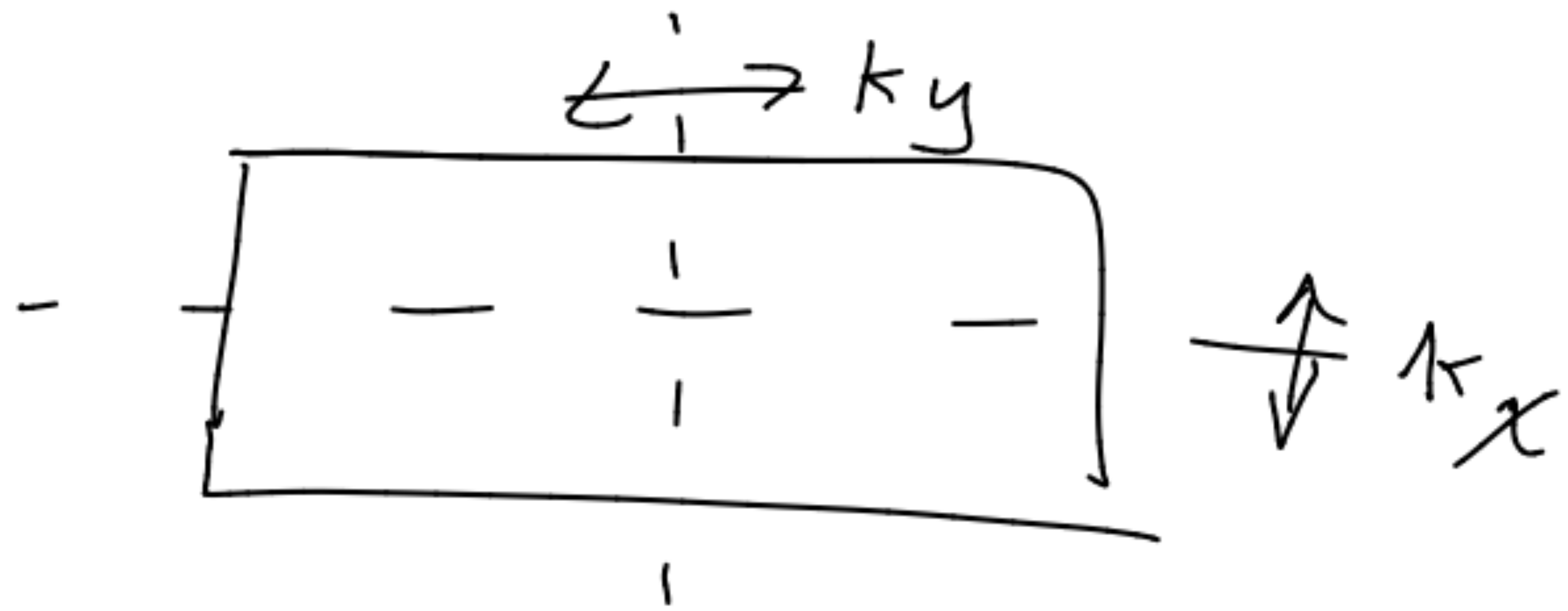
$N(Z_2) = \{\gamma \in D_6 : Z_2\gamma = \gamma Z_2\}$

Idea: Look for the largest subgroup $K \subset \Gamma$

s.t. $H \subseteq K \subseteq \Gamma$ and H is normal.

$N(Z_2) = \{e, \rho^3, k, k\rho^3\}$

$$D_2 = \{k_x, k_y\} = \{e, k_x, k_y, k_x k_y\}$$

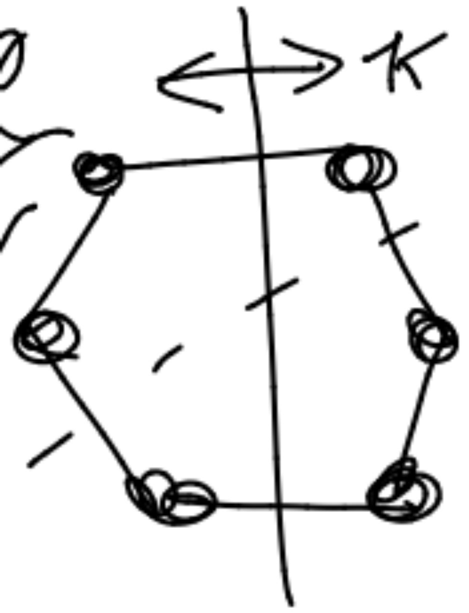


$$k_x \cdot \gamma = \gamma k_x$$

3.6 Find conjugacy classes of D_6

Suggestion: Consider the natural rep.

$$D_6 = \{e, \rho, \rho^2, \rho^3, \rho^4, \rho^5, \kappa, \kappa\rho, \kappa\rho^2, \kappa\rho^3, \kappa\rho^4, \kappa\rho^5\}$$



$M = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $\quad M_{\rho^2} \quad M_{\rho^3} \ldots$

$$R_{\frac{2\pi}{6}} = \begin{bmatrix}\cos\frac{2\pi}{6} & -\sin\frac{2\pi}{6}\\ \sin\frac{2\pi}{6} & \cos\frac{2\pi}{6}\end{bmatrix}$$

$\chi(M_e) = 2, \chi(M_\rho), \ldots$

Conjugacy classes:

$\{e\}, \{\rho, \rho^5\}, \{\rho^2, \rho^4\}, \ldots$

3.7 find irreps. of D_2 over $\mathbb{C}$

	e	k_x	k_y	$k_x k_y$
e	e	k_x	k_y	$k_x k_y$
k_x	k_x	e	$k_x k_y$	k_y
k_y	k_y	$k_x k_y$	e	k_x
$k_x k_y$	$k_x k_y$	k_y	k_x	e

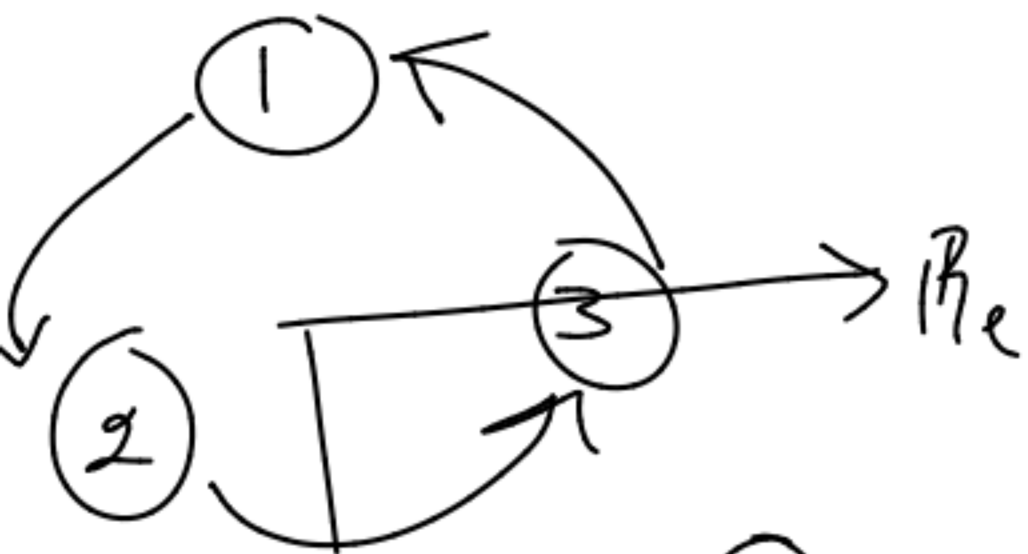
	$\{e\}$	$\{k_x\}$	$\{k_y\}$	$\{k_x k_y\}$
θ_1	1	1	1	1
θ_2	1	1	-1	-1
θ_3	1	-1	1	-1
θ_4	1	-1	-1	1

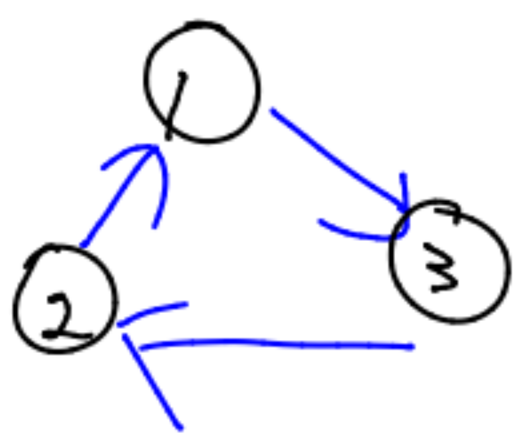
3.8 Find irrep. of $\mathbb{Z}_3$ over $\mathbb{C}$

$$\mathbb{Z}_3 = \{0, 1, 2\} \text{ or } \mathbb{Z}_3 = \{e, \rho, \rho^2\}$$

$$\rho = R_{\frac{2\pi}{3}} \text{ or } \rho = e^{\frac{2\pi}{3} i}$$

nI_n





	$\{e\}$	$\{\rho\}$	$\{\rho^2\}$
1D θ_1	1	1	1
1D θ_2	1 ρ^0	ρ^1 $e^{\frac{2\pi}{3} i}$	ρ^2 $e^{\frac{4\pi}{3} i}$
1D θ_3	1	$\rho^2 = e^{\frac{4\pi}{3} i}$	$\rho = e^{\frac{2\pi}{3} N}$

3.8 Find <u>isotypic decomp.</u> of $\mathbb{Z}_2$ on $\mathbb{R}^2$

Action of $\mathbb{Z}_2$ on $\mathbb{R}^2$ is

$$M_e = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } M_k = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Want: $\mathbb{R}^2 = V_1 \oplus V_2$, s.t. $\gamma V_i \in V_i$, $\forall \gamma \in \Gamma = \mathbb{Z}_2$

Idea: All vectors in $\mathbb{R}^2$ can be expressed as the sum of two vectors: v_1, v_2: $v_i = c_1 v_1 + c_2 v_2$

Let $B_1 = \left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{v_1}, \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{v_2} \right\}$ or $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} = B_2$

$$\left.\begin{aligned} M_e v_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \in V_1 \\ M_e v_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \in V_2 \end{aligned}\right\} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ is inv. under } e \in \mathbb{Z}_2$$

and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is also inv. under e

$$M_K \cdot V_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$M_K \cdot V_2 = \underline{\qquad\qquad}$$

Show V_1 & V_2 are both $\mathbb{Z}_2$-inv.

Let $\Gamma = S_n$

$$\mathbb{R}^n = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{1D} \oplus \underbrace{\left\{ V_1 + V_2 + \dots + V_n = 0 \right\}}_{(n-1)D}$$