e.g.) SS Bif. in a Triangle

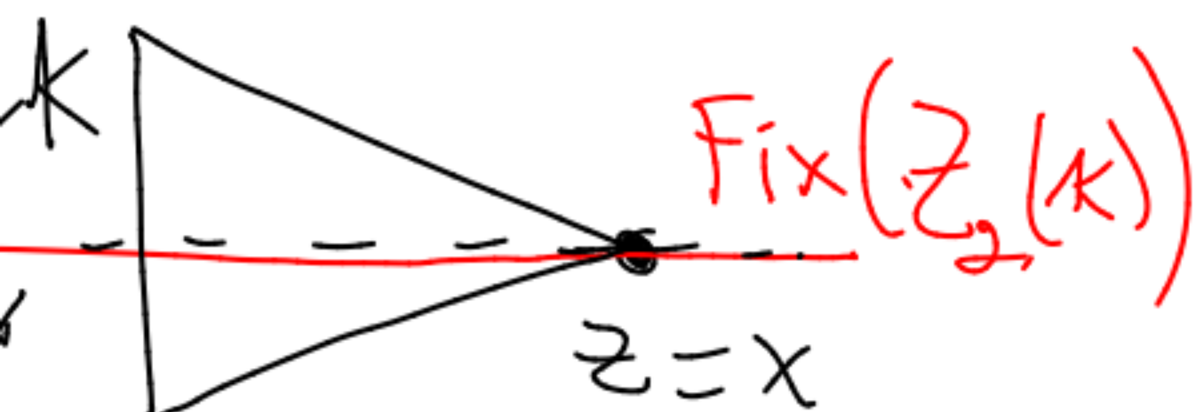
$\Gamma = D_3$, <u>Standard</u> Action

$\gamma \cdot z = e^{\gamma i} z$, $\gamma = \frac{2\pi}{3}$

$\kappa \cdot z = \bar{z}$

Amplitude Eq.

$$\Rightarrow \boxed{\frac{dz}{dt} = p(z\bar{z}, z^3 + \bar{z}^3) z + q(z\bar{z}, z^3 + \bar{z}^3)\bar{z}^2 = f(z)}$$



$\frac{dz}{dt} = f(z, \lambda) \overset{\text{set}}{=} 0 \Rightarrow$ solve for

Let $z = x + yi$, $\dot{x} = \mathrm{Re}\, z$, $\dot{y} = \underline{\quad}$

$D_3 - \mathrm{Fix}(D_3) = \{0\}$

$\uparrow$

$\mathbb{Z}_2(\kappa) - \mathrm{Fix}(\mathbb{Z}_2(\kappa)) = \mathbb{R}$

$\uparrow$

$\mathbb{I}$

$f(z, \lambda) = 0 \Big|_{z \in \mathrm{Fix}(\mathbb{Z}_2(\kappa))}$

$$\boxed{\begin{aligned} f(x, \lambda) &= p(x^2, 2x^3, \lambda)x + q(x^2, 2x^3, \lambda)x^2 \\ &= (p + qx)x \overset{\text{set}}{=} 0 \end{aligned}}$$

① $f(0, \lambda) = 0$

⊙ Nontrivial sol correspond to

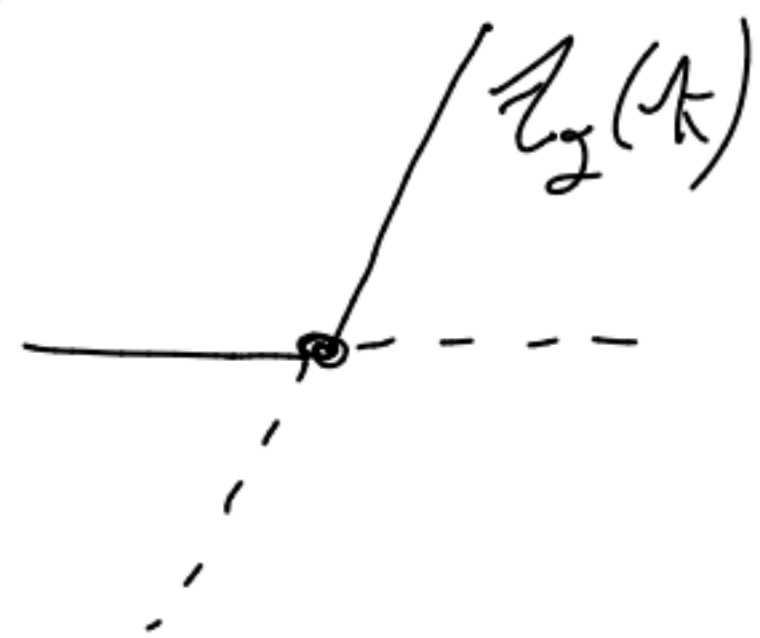
$$p + q x = 0$$

where $p(\underset{z}{0}, \underset{\bar{z}}{0}, \underset{\lambda}{0}) = 0$ since

there is a bif. of $x=0$ at $\lambda = 0$

$$\Rightarrow p_\lambda(0)\lambda + q(0)x + \dots = 0$$

$$\lambda = -\frac{q(0)}{p_\lambda(0)}x + \dots$$



Isotypic Decomposition

$$\mathbb{C} = \mathbb{R} \oplus \mathbb{R}\{i\} \equiv V_0 \oplus V_1$$

$\mathbb{Z}_2(k)$ is trivial irrep.

Nontrivial irrep.

$(df)_x$ has e-vectors 1 & i

Stability

$$(df)_z V = f_z V + f_{\bar{z}} \bar{V}$$

$\underline{\underline{n \geq 3}}$

$$f_z = p + p_u z\bar{z} + n p_v z^n + (q_u \bar{z} + n q_v z^{n-1})\bar{z}^{n-1}$$

$$f_{\bar{z}} = p_u z^2 + n p_v z \bar{z}^{n-1} + (n-1) q \bar{z}^{n-2} + (q_u z + n q_v \bar{z}^{n-1})\bar{z}^{n-1}$$

e-vals:

$$(df)\cdot 1 = f_z \cdot 1 + f_{\bar{z}} \cdot 1 = \overbrace{(f_z + f_{\bar{z}})}^{\sigma_1} \cdot 1$$

$$(df)\cdot i = f_z \cdot i - f_{\bar{z}}\, i = \underbrace{(f_z - f_{\bar{z}})}_{\sigma_2} \cdot i$$

$$\sigma_1 = f_z + f_{\bar{z}} = 2 p_u(0,0,0) x^2 + \dots$$

$$\sigma_2 = f_z - f_{\bar{z}} = -(n-1) q(0,0,0) x^{n-2} + \dots = -2qx + \dots$$

$n=3$

$$f_z = p_z z + q + q_z \bar{z}^2$$

$$= p_z x + p + q_z x^2, \quad z = x$$

$$f_{\bar{z}} = p_{\bar{z}} z + q_{\bar{z}} \bar{z}^2 + 2q\bar{z}$$

$$= p_{\bar{z}} x + q_{\bar{z}} x^2 + 2qx$$

$$\sigma_1 = f_z + f_{\bar{z}} = p + 2qx + \overbrace{(\cancel{p_z} + \cancel{p_{\bar{z}}})}^{0 \text{ at } x=0} x + (\cancel{q_{\bar{z}}} + \cancel{q_{\bar{z}}}) x^2$$

$$\sigma_2 = f_z - f_{\bar{z}} = p - 2qx + (\cancel{p_z} - \cancel{p_{\bar{z}}}) x + (\cancel{q_z} - \cancel{q_{\bar{z}}}) x^2$$

$$\sigma_1 = 2p_z x + 2q_z x^2 + 2qx + \underbrace{p}_{0}$$

$$= x(2q + 2\underbrace{p_z}_{0}) + O(x^2)$$

$$\boxed{\begin{aligned}\sigma_1 &= 2qx \\ \sigma_2 &= -2qx\end{aligned}}$$

⊙ Two e-vals of opposite sign

$\Longrightarrow$ x is a saddle

$\Gamma = D_1$