M635 Pattern Formation

Homework 3
Instructions: STAPLE ALL PAGES
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Name: ________________________________

1. (25 Pts) The Brusselator Model on a Circular Domain

Consider the Brusselator model

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa_1 \nabla^2 u + (B-1)u + A^2 v - \eta u^3 - \nu_1 (\nabla u)^2 \\ \frac{\partial v}{\partial t} &= \kappa_2 \nabla^2 v - Bu - A^2 v - \eta v^3 - \nu_2 (\nabla u)^2. \end{aligned} \tag{1}$$

Study the stability properties of the homogeneous trivial solution ($u_0 = 0, v_0 = 0$) by performing the following tasks:

1. Linearize (1) about (u_0, v_0): Set $u = u_0 + \tilde{u}$, $v = v_0 + \tilde{v}$, and write the linearized equation for $\tilde{u}, \tilde{v}$.
2. Let $\tilde{u}(x, y, t) = \hat{u}\Psi_{nm}$, $\tilde{v}(x, y, t) = \hat{v}\Psi_{nm}$, where $\Psi_{nm}(r, \phi) = J_n(\alpha_{nm} r/R) e^{in\phi}$. Substitute and solve the resulting eigenvalue problem for σ.
3. Compute marginal stability curves $\sigma_n = 0$ and study the behavior of the perturbation for various values of n in the (B, R) plane.
4. Use MATLAB or an equivalent software to plot a few marginal stability curves B_{nm} as a function of radius R and also plot a few representative examples of Fourier-Bessel modes.

2. (25 Pts) Swift-Hohenberg Model on a Finite Line

The Swift-Hohenberg equation (1977) was originally proposed as a simplified model of convective instability in a one-dimensional system. It takes the form

$$\frac{\partial u}{\partial t} = \left[\mu - (\nabla^2 + k_c^2)^2\right] u - u^3. \tag{2}$$

Assume $k_c^2 = 1$ and the following boundary conditions

$$u = \frac{\partial^2 u}{\partial x^2}, \qquad x = 0,\ L.$$

Study the stability properties of the homogeneous trivial solution $u_0 = 0$ by performing the following tasks:

1. Linearize (2) about $u = u_0$: Set $u = u_0 + \tilde{u}$ and write the linearized equation for $\tilde{u}$.

2. Solve linearized equation for $\tilde{u}$ using separation of variables: Set $\tilde{u}(x,t) = G(t)\phi(x)$, substitute and divide across by $G(t)\phi(x)$. Set separated variables to a common constant σ and then solve the resulting eigenvalue problems for $G(t)$ and $\phi(x)$.

3. Compute marginal stability curves $\sigma_n = 0$ and study the behavior of the perturbation for various values of n in the (μ, L) plane.

3. (50 Pts) Review of Article on Cellular Pattern Formation