M636 MATHEMATICAL MODELING

Homework – Fall 2015

Name: ________________________________

Instructions

1. Homework is considered, strictly, individual: all work should be your own, carried out individually, not in a group. Any form of cheating (including copying someone else work or allowing to be copied) will result in an "F" in the final grade of the course and a referral to the Dean's office for further action.
2. Show all analytical work.
3. Graphical and/or computational work can also be included in support of the analysis. But unless otherwise stated in the problem, answers based purely on graphical and/or computational work are considered incomplete.

I, ________________________, pledge that this homework is completely my own work, and that I did not consult, take, copy, borrow or steal any portions from any other person. I understand that if I violate this honesty pledge, I am subject to disciplinary action pursuant to the appropriate sections of the San Diego State University Policies.

1. The "Brusselator" model, proposed by Prigogene and Lefever (1968), describes the evolution of two coupled, diffusive spatio-temporal fields $u(\mathbf{x},t)$ and $v(\mathbf{x},t)$ through

$$\begin{aligned} \frac{\partial u}{\partial t} &= \kappa_1 \nabla^2 u + (B-1)u + A^2 v - \eta u^3 - \nu_1 (\nabla u)^2, \\ \frac{\partial v}{\partial t} &= \kappa_2 \nabla^2 v - Bu - A^2 v - \eta u^3 - \nu_2 (\nabla v)^2, \end{aligned} \tag{1}$$

where κ_1 and κ_2 are diffusion coefficients, A, B, η, ν_1 and ν_2 are additional nonnegative parameters.

(a) Determine the linearized system about the trivial equilibrium point $(u_0, v_0) = (0, 0)$.

(b) Characterize the stability of the trivial equilibrium point. Find the characteristic equation and determine the eigenvalues.

(c) Derive a formula for the marginal stability curves in response to perturbations of the form $(\delta u, \delta v)\Psi_{nm}$, where $\Psi_{nm}(r,\theta) = J_n(\alpha_{nm}\frac{r}{R})e^{in\theta}$ are Fourier-Bessel modes. Write the formula in terms of B_{nm} as a function of $|\vec{k}|^2 = (\frac{\alpha_{nm}}{R})^2$.

(d) Let $\kappa_1 = 0.2$, $\kappa_2 = 2.0$, $A = 5.0$, $\alpha_{12} = 7.01559$, $\alpha_{31} = 6.38016$, and $\alpha_{41} = 7.58834$. In ONE graph (ONE page), plot B_{12}, B_{31} and B_{41} as a function of R. Scale the plot to a window of size $R \in [2, 3] \times B \in [6.6, 6.9]$. In ONE graph (ONE page), plot $\Psi_{12}(r,\theta)$, $\Psi_{31}(r,\theta)$, $\Psi_{41}(r,\theta)$. Note: Failure to follow these instructions will affect the amount of partial credit.

(e) Explain the relation between the marginal stability curves B_{nm} and the Fourier-Bessel modes Ψ_{nm} drawn in part (d).